NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF NATURAL AND APPLIED SCIENCES

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	**LEVEL:** 7
COURSE NAME: QUANTUM PHYSICS	**COURSE CODE:** QPH702S
SESSION: JANUARY 2020	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	**Prof Dipti R. Sahu**
MODERATOR:	**Dr Habatwa V. Mweene**

INSTRUCTIONS
1. Answer any **five** questions. 2. Write clearly and neatly. 3. Number the answers clearly.

PERMISSIBLE MATERIALS

Non-programmable Calculators

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1 [20]

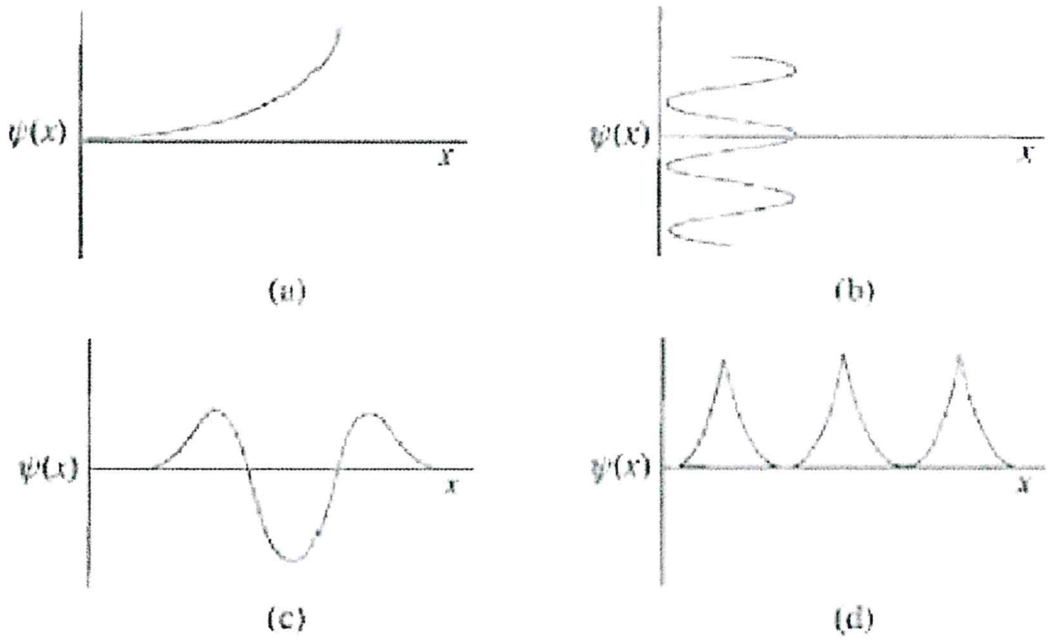
The wave function of a particle moving in the *x*-dimension is

$$\psi(x) = \begin{cases} Nx(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

(a) Normalize the wave function (5)
(b) Determine the expectation value of x (5)
(c) Calculate $<p_x>$, $<p_x^2>$ and Δp_x (10)

Question 2 **[20]**

2.1 Which of the wave functions shown in the figure are well behaved? Give reasons for your answers. (5)



2.2 An atom of mass *m* is attached to another by a one-dimensional harmonic oscillator having a potential energy with spring constant *k*, defined so that *F=-kx.*

(a) Write down the one-dimensional time-independent Schrödinger equation for *Ψ(x)* with the harmonic oscillator potential. (5)

(b) Draw a sketch of the wave function Ψ(x) and the probability density P(x) for the two lowest energy states. (5)

(c) The wave function for the ground state is

$$\psi_0(x) = C_0 e^{-\alpha^2 x^2/2}$$

By direct substitution, find α and the energy corresponding to this state. (10)

Question 3 [20]

3.1 An electron has a kinetic energy of 12.0 eV. The electron is incident upon a rectangular barrier of height 20.0 eV and thickness 1.00 nm. By what factor would the electron's probability of tunneling through the barrier increase assuming that the electron absorbs all the energy of a photon with wavelength 546 nm (green light)? (5)

3. 2 The potential function *V(x)* of the problem is given by

$$V(x) = \begin{cases} V_o & x > 0 \\ 0 & x < 0 \end{cases}$$

where V_o is a constant potential energy.

(a) Sketch the graph of this function (2)

(b) Find the wave function for $\varepsilon < V_o$ where ε is the incident particle energy and interpret the results. (13)

Question 4 **[20]**

4.1 What are the kinetic, potential and Hamiltonian operators for the hydrogen atom? Write the Schrodinger equation for the H-atom. (5)

4.2 Show, for Hermitian operators *A* and *B*, that the product *AB* is a Hermitian operator if and only if *A* and *B* commute. (5)

4.3 Show explicitly in Cartesian coordinates(x, y, z) that the operators ∇^2 and L_z commute, i.e., $[\nabla^2, L_z] = 0$. (10)

Question 5 **[20]**

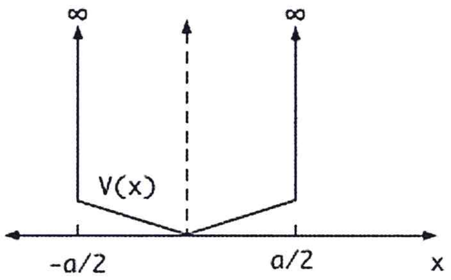
5.1 What are the Pauli spin matrices and to what value of spin they correspond? Write them down. (5)

5.2 For each Pauli matrix, find its eigenvalues, and the components of its normalized eigenvectors in the basis of the eigenstates of S_z. (10)

5.3 Evaluate the matrix of L_y for $l = 1$. Why is the matrix not diagonal? (5)

Question 6 **[20]**

6.1 Consider an infinite well for which the bottom is not flat, as sketched here. If the slope is small, the potential $V = \varepsilon\ |x|\ /\ a$ may be considered as a perturbation on the square-well potential over $-a/2 \le x \le a/2$.



(a) Calculate the ground-state energy correct to first order. (5)

(b) Calculate the energy of the first excited state correct to first order. (5)

(c) Calculate the wave function in the ground state, correct to first order in perturbation theory. (do *not* evaluate integrals you encounter here). (5)

(d) At what value of εdoes perturbation theory break down? Justify your answer. (5)

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Useful Standard Integrals

Plank constant = 6.63×10^{-34}Js

$$\int_{-\infty}^{\infty} e^{-y^2}\, dy = \sqrt{\pi}$$

Speed of light = 3×10^8 m/s

$$\int_{-\infty}^{\infty} y^n e^{-y^2}\, dy = \frac{\sqrt{\pi}}{n}; \quad n \quad \text{even}$$

$$0; \quad n \text{ odd}$$

Mass of electron = 9.11×10^{-31} kg

$$\int_{-\infty}^{\infty} e^{-\alpha y^2} e^{-\beta y}\, dy = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{\frac{\beta^2}{4\alpha}}$$

$$R_{nl}(r) = -\left(\frac{2}{na_0}\right)^{3/2} \sqrt{\frac{(n-l-1)!}{2n[(n+l)!]^3}} \left(\frac{2r}{na_0}\right)^l e^{-r/na_0} L_{n+l}^{2l+1}\left(\frac{2r}{na_0}\right)$$

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